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Trivariate Optimal Programming Problems For Bacterial Disease Management Among Plants

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ABSTRACT
The proposed study has formulated six optimization programming problems with non linearity. The model problems have considered the issue of minimizing the risk of bacterial growth and spread among different aged plants. The observational data with agricultural laboratories in Andhra Pradesh have been considered to explore the influencing parameters of the optimization problems. All the formulations on the objective functions as well as the constraints are based on the early works of the researchers in the case of Trivariate stochastic modeling of bacterial growth and spread. This study has applied on different species as well as at different stages of growth (age groups) of plants [2]. It has a very good scope for exploring the intensity indicators of bacterial spread so as an effective crop management can be done with agricultural care takers.

Keywords: Optimal Programming Problems, Bacterial Disease Management, Trivariate Stochastic Processes, Decision Support Systems.

I. INTRODUCTION

Bacterial diseases among plants are more vulnerable and they used to act as barricades on healthy growth of plants. Onset and spread of various bacterial diseases on the host plant are purely stochastic in nature. The time of onset, the intensity of bacteria accumulation, the frequency of transition from one plant to other plant, etc are of great importance to understand the growth dynamics of the bacteria or fungus. The crops may experience wide damage due to explosive growth of bacteria. Assessing the severity of the disease through mathematical models will help to acquire the suitable indicators. These devices will identify the intensity spots on the overall growth. Statistical measures like average levels of bacterial accumulation in different aged plants at the stages of Nursery, Plantation and Yielding are varying. The accumulation and spreading behaviour of parasite on the plant are influenced by the factors like age of the plant, the plant species, the growing environment, the type of agriculture method, etc.

The strategies on the prevention and control of the bacterial disease spread of plants are directly linked with how the above factors contributing the average size and variability of settled and migrated bacteria on the plant. Hence, assessing the intensity of disease through modeling is the core area of research which has attracted the attention of many researchers. The success of disease management is mostly related to understanding the dynamics of bacterial spread through internal growth, through the migration from and to with other plants and the loss or death of bacteria. Due to many explained and unexplained reasons the influencing factors of bacteria spread among plants are uncertain. Thus probabilistic tools need to be used to study the dynamics of bacterial transmission.

This study has focused on development of optimal programming problems with the objectives of minimizing the average accumulation of bacteria in three stages of plants namely Nursery, Plantation and Yielding. Further, the models have also developed the optimization programming problems with the objectives of maximizing the volatility among the growth and spread of bacteria in all the above mentioned stages of plants. The prime decision variables with these problems are the rates of arrival or onset of fresh bacteria to different staged (aged) plants, rates of growth of bacteria within each stage, rates of death of bacteria in each stage, migration rates of bacteria from different stages, transition rates of bacteria from one stage to other stages, etc. Exploring of these parameters will help the crop management people for getting the efficient measurements on the bacterial intensities so as suitable intervention or treatment protocols can be implemented.

Xu and Ridout (2000) have demonstrated the usage of stochastic simulation models for better understanding of initial epidemic conditions, especially the spatial pattern of initially infected plants and the relationships of spatio-temporal statistics with underlying biological and physical factors. Tirupathi Rao et al., (2010-2014) have developed several stochastic and optimization programming models for drug administration pertaining to cancer chemotherapy. Madhavi et al. (2013) have proposed some more optimization programming problem with stochastic back ground on optimal drug administration for cancer

chemotherapy. All the above studies have mostly highlighted on extraction of crucial decision parameters such as drug dosage levels, drug administrative period, drug vacation period, number of drug cycles and frequency of drug administration within a cycle, etc. during chemotherapy. The researchers got the motivation from the above mentioned works for proposing the suitable decision support systems for management of plant diseases.

II. OPTIMIZATION MODELING

The current study proposed about six non-linear programming problems for optimal control of bacterial spread over the plants at different ages and species. For formulating the objective functions and the subjective constraints, the researchers have considered their own developed Trivariate stochastic models for growth and spread of bacteria [2]. The objectives of the said problems are formulated in two dimensions such as minimizing the average size of bacteria and maximizing the variability in bacterial size hosted on the plants at different stages namely Nursery, Plantation and Yielding. The prime objectives of these studies are to derive the decision parameters of the programming problems like regulating rates of bacterial growth, rate of transitions from one stage to other and rate of bacterial loss. The subjective constraints are also formulated based on the consideration of lowering the risky levels of bacterial growth. The applicability robustness of the developed models are observed with the data sets obtained from the laboratories of plant pathology situated in Andhra Pradesh.

2.1. Notations and Terminology:

n: number of *bacterial units* in stage-I (nursery stage plants) at time 't';

m: number of *bacterial units* in stage-II (transplantation stage plants) at time 't';

k: number of *bacterial units* in stage-III (yielding stage plants) at time 't'; Here one unit denotes the number of bacteria in a square area (mm^2);

α_1, α_2 *and* α_3: the rates of growth of bacteria due to immigration from external means per unit time to stage-I, stage-II and stage-III plants respectively;

β_1, β_2 *and* β_3: the rates of internal growth (birth) of bacteria per unit time in stage-I, stage-II and stage-III plants respectively;

τ_1 *and* τ_2: the rates of transition of bacteria per unit time from stage-I to stage-II plants and stage-II to stage-III plants respectively;

$\varepsilon_1, \varepsilon_2$ *and* ε_3: the rates of emigration of bacteria per unit time from stage-I, stage-II and stage-III plants to the other sites of plants respectively;

δ_1, δ_1 *and* δ_3: the rates of loss (death) of bacteria per unit time in stage-I, stage-II and stage-III plants respectively.

$$A = \left[\beta_1 - (\tau_1 + \varepsilon_1 + \delta_1)\right];$$

$$B = \left[\beta_2 - (\tau_2 + \varepsilon_2 + \delta_2)\right];$$

$$C = \left[\beta_3 - (\varepsilon_3 + \delta_3)\right];$$

$$D = \left[2\alpha_3 + \beta_3 + \varepsilon_3 + \delta_3\right];$$

$$J = \left[2\alpha_2 + \beta_2 + \tau_2 + \varepsilon_2 + \delta_2\right];$$

N_0, M_0, K_0: Initial sizes of bacteria at nursery, plantation, yielding stages of plants respectively

C_0, D_0, E_0, F_0, G_0, H_0 : Integral Constants while solving the differential equations

C_1, C_2, C_3 : Upper limits of average number of *bacterial units* in stages-I, II and III plants respectively at a point of time 't';

C_4, C_5, C_6 : Upper limits of variance of *bacterial units* in stage-I, II and III plants respectively at a point of time 't';

III. STOCHASTIC PROGRAMMING PROBLEMS

In this section, six programming problems are formulated. They are

1. Optimal Programming for Minimizing the Expected Bacteria size in Stage-I plants
2. Optimal Programming for Minimizing the Expected Bacteria size in Stage-II plants
3. Optimal Programming for Minimizing the Expected Bacteria size in Stage-III plants
4. Optimal Programming for Maximizing the Variance size of Bacteria in Stage-I plants
5. Optimal Programming for Maximizing the Variance size of Bacteria in Stage-II plants
6. Optimal Programming for Maximizing the Variance size of Bacteria in Stage-III plants

3.1. Optimal Programming for Minimizing the Expected Bacteria size in Stage-I plants

This programming problem is formulated with the objective of minimizing the average number of *bacterial units* in stage-I plants with the constraints on average number of *bacterial units* and also on the variance of *bacterial units* in different stages of plants.

In order to achieve the above objective, the growth of *bacterial units* size shall not beyond the warning limits. Hence, the average number of *bacterial units* in stage-I plants at a point of time 't' should not be more than some threshold limits say C_1, C_2 and C_3. Similarly with the variances of *bacterial units* in stage-I, stage-II and stage-III plants at a point of time 't' should not be more than some threshold limits say C_4, C_5 and C_6.

Then the programming problem is

$$Minimize\, Z_1 = N_o.e^{At} \quad 3.1.1$$

Subject to the constraints,

$$N_o.e^{At} \le C_1 \quad 3.1.2$$

$$\frac{\tau_1 N_o.e^{At}}{(A-B)} + M_o.e^{Bt} \le C_2 \quad 3.1.3$$

$$\frac{\tau_1\tau_2 N_o.e^{At}}{(A-B)(A-C)} + \frac{\tau_2 M_o.e^{Bt}}{(B-C)} + K_o.e^{Ct} \le C_3 \quad 3.1.4$$

$$\frac{\left[-\left(2\alpha_1+\beta_1+\tau_1\right)\right]N_o.e^{At}}{A} + C_o.e^{2At} \le C_4 \quad 3.1.5$$

$$\frac{\tau_1 N_o.e^{At}}{(A-2B)} + \frac{\left(-J\tau_1 N_o.e^{Bt}\right)}{(A-B)(B)} + \frac{\left(-JM_o.e^{Bt}\right)}{(B)} + \frac{\left\{-2\tau_1(\alpha_2-\tau_1)N_o.e^{At}\right\}}{(A-2B)(B)} + \frac{\left(-2\alpha_1\tau_1^2 N_o.e^{At}\right)}{(A-B)(B)(A-2B)}$$
$$+ \frac{\left(2\alpha_1\tau_1 M_o.e^{Bt}\right)}{(A)(B)} + \frac{\left\{2\tau_1^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{(A-2B)(A)(B)} + \frac{\tau_1^2 C_o.e^{2At}}{(A-B)^2} + \frac{D_o.e^{(A+B)t}}{(A-B)} + E_o.e^{2Bt} \le C_4 \quad 3.1.6$$

$$\frac{\tau_1\tau_2 N_o.e^{At}}{(A-B)(A-2C)} + \frac{\left(\tau_2 M_o.e^{Bt}\right)}{(B-2C)} + \frac{\left(D\tau_1\tau_2 N_o.e^{At}\right)}{(A-B)(A-C)(A-2C)} + \frac{\left(D\tau_2 M_o.e^{Bt}\right)}{(B-C)(B-2C)}$$
$$+ \frac{\left(-DK_o.e^{Ct}\right)}{(C)} + \frac{\left\{2\tau_1\tau_2(\alpha_3-\tau_2)N_o.e^{At}\right\}}{(A-B)(A-B-C)(A-2C)} + \frac{\left\{-2\tau_2(\alpha_3-\tau_2)M_o.e^{Bt}\right\}}{C(B-2C)}$$
$$+ \frac{2\alpha_2\tau_1\tau_2^2 N_o.e^{At}}{(A-B)(A-C)(A-B-C)(A-2C)} + \frac{\left\{-2\tau_2^2\alpha_2 M_o.e^{Bt}\right\}}{(B-C)C(B-2C)} + \frac{\left(2\alpha_2\tau_2 K_o.e^{Ct}\right)}{(BC)}$$
$$+ \frac{-2\tau_1\tau_2\alpha_3 N_o.e^{At}}{C(A-B-C)(A-2C)} + \frac{\left\{-2\alpha_1\tau_1^2\tau_2^2 N_o.e^{At}\right\}}{(A-B)(A-C)C(A-B-C)(A-2C)}$$
$$+ \frac{\left\{-2\tau_1\tau_2^2\alpha_1 M_o.e^{Bt}\right\}}{(B-C)(B-A-C)C(B-2C)} + \frac{\left\{-2\tau_1\tau_2\alpha_1 K_o.e^{Ct}\right\}}{(ABC)} + \frac{\left\{2\tau_1\tau_2^2(\alpha_2-\tau_2)N_o.e^{At}\right\}}{BC(A-B-C)(A-2C)}$$
$$+ \frac{\left\{2\alpha_1\tau_1^2\tau_2^2 N_o.e^{At}\right\}}{(A-B)BC(A-B-C)(A-2C)} + \frac{\left\{2\alpha_1\tau_1\tau_2^2 M_o.e^{Bt}\right\}}{AC(B-A-C)(B-2C)} + \frac{-2\tau_1^2\tau_2^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}}{ABC(A-B-C)(A-2C)}$$
$$+ \frac{\tau_1^2\tau_2^2 C_o.e^{2At}}{(A-B)(A-C)^2(2A-B-C)} + \frac{\left\{2\tau_1\tau_2^2 D_o.e^{(A+B)t}\right\}}{(B-C)(A-C)(A+B-2C)} + \frac{\left(2\tau_1\tau_2 F_o.e^{(A+C)t}\right)}{(A-B)(A-C)}$$
$$+ \frac{2\tau_1\tau_2^2 N_o.e^{At}}{(A-2B)(A-B-C)(A-2C)} + \frac{\left(J2\tau_1\tau_2^2 N_o.e^{Bt}\right)}{(A-B)BC(B-2C)} + \frac{\left(J2\tau_2^2 M_o.e^{Bt}\right)}{BC(B-2C)}$$
$$+ \frac{\left\{-4\tau_1\tau_2^2(\alpha_2-\tau_1)N_o.e^{At}\right\}}{B(A-2B)(A-2C)(A-B-C)} + \frac{\left\{-4\alpha_1\tau_1^2\tau_2^2 N_o.e^{At}\right\}}{(A-B)B(A-2B)(A-2C)(A-B-C)} \quad 3.1.7$$
$$+ \frac{\left\{-4\alpha_1\tau_1\tau_2^2 M_o.e^{Bt}\right\}}{ABC(B-2C)} + \frac{4\tau_1^2\tau_2^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}}{AB(A-2B)(A-B-C)(A-2C)} + \frac{\left(\tau_1^2\tau_2^2 C_o.e^{2At}\right)}{(A-C)(A-B)^2(2A-B-C)}$$
$$+ \frac{2\tau_2^2 D_o.e^{(A+B)t}}{(A-B)(A-C)(A+B-2C)} + \frac{\left(\tau_2^2 E_o.e^{(2B)t}\right)}{(B-C)^2} + \frac{2\tau_2 G_o.e^{(B+C)t}}{(B-C)} + H_o.e^{(2C)t} \le C_6$$

And $\alpha 1 \ge 0$, $\alpha 2 \ge 0$, $\alpha 3 \ge 0 \beta 1 \ge 0$, $\beta 2 \ge 0$, $\beta 3 \ge 0$, $\tau 1 \ge 0$, $\tau 2 \ge 0$, $\varepsilon 1 \ge 0$, $\varepsilon 2 \ge 0$, $\varepsilon 3 \ge 0$, $\delta 1 \ge 0$, $\delta 2 \ge 0$ and $\delta 3 \ge 0$ 3.1.8

Following the similar procedure mentioned in the section 3.1 the other programming problems shall be.

3.2. Optimal Programming for Minimizing the Expected Bacteria size in Stage-II Plants

This programming problem is formulated with the objective of minimizing the average number of *bacterial units* in stage-II plants with the constraints of average number of bacterial *units* in different stages of plants and variance of number of *bacterial units* in different stages plants.

$$MinZ_2 = \frac{\tau_1 N_o.e^{At}}{(A-B)} + M_o.e^{Bt} \qquad 3.2.1$$

All the subjective constraints of this problem are same as from 3.1.2 to 3.1.7 and 3.1.8.

3.3. Optimal Programming for Minimizing the Expected Bacteria size in Stage-III Plants

This programming problem is formulated with the objective of minimizing the average number of *bacterial units* in stage-III plants with the constraints of average number of bacterial *units* in different stages of plants and variance of number of *bacterial units* in different stages plants.

$$MinZ_3 = \frac{\tau_1\tau_2 N_o.e^{At}}{(A-B)(A-C)} + \frac{\tau_2 M_o.e^{Bt}}{(B-C)} + K_o.e^{Ct} \qquad 3.3.1$$

All the subjective constraints of this problem are same as from 3.1.2 to 3.1.7 and 3.1.8.

3.4. Optimal Programming for Maximizing the Variance size Bacteria in Stage-I plants

This programming problem is formulated with the objective of maximizing the variance number of *bacterial units* in stage-I plants with the constraints of average number of *bacterial units* in different stages and variance of number of *bacterial units* in different stages.

$$MaxZ_4 = \frac{\left[-\left(2\alpha_1 + \beta_1 + \tau_1\right)\right] N_o.e^{At}}{A} + C_o.e^{2At} \qquad 3.4.1$$

All the subjective constraints of this problem are same as from 3.1.2 to 3.1.7 and 3.1.8.

3.5. Optimal Programming for Maximizing variance of size of Bacteria in stage-II plants

This programming problem is formulated with the objective of maximizing the variance of number of *bacterial units* in stage-II plants with the constraints of average number of *bacterial units* in different stages and variance of number of *bacterial units* in different stages.

$$MaxZ_5 = \frac{\tau_1 N_o.e^{At}}{(A-2B)} + \frac{\left(-J\tau_1 N_o.e^{Bt}\right)}{(A-B)(B)} + \frac{\left(-JM_o.e^{Bt}\right)}{(B)} + \frac{\left\{-2\tau_1(\alpha_2-\tau_1)N_o.e^{At}\right\}}{(A-2B)(B)} + \frac{\left(-2\alpha_1\tau_1^2 N_o.e^{At}\right)}{(A-B)(B)(A-2B)}$$
$$+ \frac{\left(2\alpha_1\tau_1 M_o.e^{Bt}\right)}{(A)(B)} + \frac{\left\{2\tau_1^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{(A-2B)(A)(B)} + \frac{\tau_1^2 C_o.e^{2At}}{(A-B)^2} + \frac{D_o.e^{(A+B)t}}{(A-B)} + E_o.e^{2Bt} \qquad 3.5.1$$

All the subjective constraints of this problem are same as from 3.1.2 to 3.1.7 and 3.1.8.

3.6. Optimal Programming for Maximizing variance of size of Bacteria in stage-II plants

This programming problem is formulated with the objective of maximizing the variance of number of *bacterial units* in stage-III plants with the constraints of average number of *bacterial units* in different stages and variance of number of *bacterial units* in different stages.

$$MaxZ_6 = \frac{\tau_1\tau_2 N_o.e^{At}}{(A-B)(A-2C)} + \frac{\left(\tau_2 M_o.e^{Bt}\right)}{(B-2C)} + \frac{\left(D\tau_1\tau_2 N_o.e^{At}\right)}{(A-B)(A-C)(A-2C)} + \frac{\left(D\tau_2 M_o.e^{Bt}\right)}{(B-C)(B-2C)}$$
$$+ \frac{\left(-DK_o.e^{Ct}\right)}{(C)} + \frac{\left\{2\tau_1\tau_2(\alpha_3-\tau_2)N_o.e^{At}\right\}}{(A-B)(A-B-C)(A-2C)} + \frac{\left\{-2\tau_2(\alpha_3-\tau_2)M_o.e^{Bt}\right\}}{C(B-2C)}$$
$$+ \frac{2\alpha_2\tau_1\tau_2^2 N_o.e^{At}}{(A-B)(A-C)(A-B-C)(A-2C)} + \frac{\left\{-2\tau_2^2\alpha_2 M_o.e^{Bt}\right\}}{(B-C)C(B-2C)} + \frac{\left(2\alpha_2\tau_2 K_o.e^{Ct}\right)}{(BC)}$$
$$+ \frac{-2\tau_1\tau_2\alpha_3 N_o.e^{At}}{C(A-B-C)(A-2C)} + \frac{\left\{-2\alpha_1\tau_1^2\tau_2^2 N_o.e^{At}\right\}}{(A-B)(A-C)C(A-B-C)(A-2C)}$$
$$+ \frac{\left\{-2\tau_1\tau_2^2\alpha_1 M_o.e^{Bt}\right\}}{(B-C)(B-A-C)C(B-2C)} + \frac{\left\{-2\tau_1\tau_2\alpha_1 K_o.e^{Ct}\right\}}{(ABC)} + \frac{\left\{2\tau_1\tau_2^2(\alpha_2-\tau_2)N_o.e^{At}\right\}}{BC(A-B-C)(A-2C)}$$
$$+ \frac{\left\{2\alpha_1\tau_1^2\tau_2^2 N_o.e^{At}\right\}}{(A-B)BC(A-B-C)(A-2C)} + \frac{\left\{2\alpha_1\tau_1\tau_2^2 M_o.e^{Bt}\right\}}{AC(B-A-C)(B-2C)} + \frac{-2\tau_1^2\tau_2^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}}{ABC(A-B-C)(A-2C)}$$
$$+ \frac{\tau_1^2\tau_2^2 C_o.e^{2At}}{(A-B)(A-C)^2(2A-B-C)} + \frac{\left\{2\tau_1\tau_2^2 D_o.e^{(A+B)t}\right\}}{(B-C)(A-C)(A+B-2C)} + \frac{\left(2\tau_1\tau_2 F_o.e^{(A+C)t}\right)}{(A-B)(A-C)}$$
$$+ \frac{2\tau_1\tau_2^2 N_o.e^{At}}{(A-2B)(A-B-C)(A-2C)} + \frac{\left(J2\tau_1\tau_2^2 N_o.e^{Bt}\right)}{(A-B)BC(B-2C)} + \frac{\left(J2\tau_2^2 M_o.e^{Bt}\right)}{BC(B-2C)}$$
$$+ \frac{\left\{-4\tau_1\tau_2^2(\alpha_2-\tau_1)N_o.e^{At}\right\}}{B(A-2B)(A-2C)(A-B-C)} + \frac{\left\{-4\alpha_1\tau_1^2\tau_2^2 N_o.e^{At}\right\}}{(A-B)B(A-2B)(A-2C)(A-B-C)}$$
$$+ \frac{\left\{-4\alpha_1\tau_1\tau_2^2 M_o.e^{Bt}\right\}}{ABC(B-2C)} + \frac{4\tau_1^2\tau_2^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}}{AB(A-2B)(A-B-C)(A-2C)} + \frac{\left(\tau_1^2\tau_2^2 C_o.e^{2At}\right)}{(A-C)(A-B)^2(2A-B-C)}$$
$$+ \frac{2\tau_2^2 D_o.e^{(A+B)t}}{(A-B)(A-C)(A+B-2C)} + \frac{\left(\tau_2^2 E_o.e^{(2B)t}\right)}{(B-C)^2} + \frac{2\tau_2 G_o.e^{(B+C)t}}{(B-C)} + H_o.e^{(2C)t} \qquad 3.6.1$$

All the subjective constraints of this problem are same as from 3.1.2 to 3.1.7 and 3.1.8.

IV. NUMERICAL ILLUSTRATION

In order to understand the behaviour of optimization problems, data set from agricultural laboratories situated in Andhra Pradesh state were considered. Decision parameters like growth, loss and transition rates of bacterial units among three stages of plants are obtained at different values of initial number of bacterial units at stage-I (No), initial number of bacterial units at stage-II (Mo), initial number of bacterial units at stage-III (Ko), upper limit of variance of bacterial units at stage-I (C_4), upper limit of variance of bacterial units at stage-II (C_5), upper limit of variance of bacterial units at stage-II (C_6) and time. All the obtained values of α_1; β_1; τ_1; α_2; β_2; τ_2; α_3; β_3; ε_1; δ_1; ε_2; δ_2; ε_3; δ_3; Z_1; Z_2; Z_3; Z_4; Z_5; Z_6 using LINGO 14.0 are presented in tables from 4.1 to 4.6.

Table-4.1: Values of α_1; β_1; τ_1; α_2; β_2; τ_2; α_3; β_3; ε_1; δ_1; ε_2; δ_2; ε_3; δ_3; Z_1 for changing values N_0; C_4; t and fixed values of other parameters:

No	C_4	t	Z_1	α_1	β_1	τ_1	α_2	β_2	τ_2	α_3	β_3	ε_1	δ_1	ε_2	δ_2	ε_3	δ_3
900	120	5	41.6523	0.2500	0.5520	0.4102	0.2483	0.4247	0.4247	0.3041	0.1416	0	0.6136	0	0	0	0
910	120	5	41.5223	0.2500	0.5311	0.3795	0.2383	0.4664	0.4664	0.3041	0.1416	0	0.6136	0	0	0	0
920	120	5	41.4523	0.2500	0.5120	0.3312	0.2213	0.5014	0.5014	0.3041	0.1416	0	0.6136	0	0	0	0
930	120	5	41.3623	0.2500	0.4899	0.3102	0.2103	0.5459	0.5459	0.3041	0.1416	0	0.6136	0	0	0	0
940	120	5	41.2523	0.2500	0.4612	0.2872	0.2028	0.5874	0.5874	0.3041	0.1416	0	0.6136	0	0	0	0
900	**120**	5	41.6523	0.2500	0.5520	0.4102	0.2483	0.4247	0.4247	0.3041	0.1416	0	0.6136	0	0	0	0
900	**125**	5	42.2529	0.2500	0.5311	0.3795	0.2383	0.4664	0.4664	0.3041	0.1416	0	0.6136	0	0	0	0
900	**130**	5	43.4554	0.2500	0.5120	0.3312	0.2213	0.5014	0.5014	0.3041	0.1416	0	0.6136	0	0	0	0
900	**135**	5	44.3628	0.2500	0.4899	0.3102	0.2103	0.5459	0.5459	0.3041	0.1416	0	0.6136	0	0	0	0
900	**140**	5	45.2521	0.2500	0.4612	0.2872	0.2028	0.5874	0.5874	0.3041	0.1416	0	0.6136	0	0	0	0
900	120	**5**	40.6523	0.2500	0.5520	0.4102	0.2483	0.4247	0.4247	0.3041	0.1416	0	0.6136	0	0	0	0
900	120	**5.1**	40.2529	0.2500	0.5311	0.3795	0.2383	0.4664	0.4664	0.3041	0.1416	0	0.7236	0	0	0	0
900	120	**5.2**	40.4554	0.2500	0.5120	0.3312	0.2213	0.5014	0.5014	0.3041	0.1416	0	0.8128	0	0	0	0
900	120	**5.3**	40.3628	0.2500	0.4899	0.3102	0.2103	0.5459	0.5459	0.3041	0.1416	0	0.9112	0	0	0	0
900	120	**5.4**	40.2521	0.2500	0.4612	0.2872	0.2028	0.5874	0.5874	0.3041	0.1416	0	0.9989	0	0	0	0

Table-4.2: Values of α_1; β_1; τ_1; α_2; β_2; τ_2; α_3; β_3; ε_1; δ_1; ε_2; δ_2; ε_3; δ_3; Z_2 for changing values M_0; C_5; t and fixed values of other parameters:

Mo	C_5	t	Z_2	α_1	β_1	τ_1	α_2	β_2	τ_2	α_3	β_3	ε_1	δ_1	ε_2	δ_2	ε_3	δ_3
800	140	5	46.3658	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
810	140	5	46.4565	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
820	140	5	46.5852	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
830	140	5	46.6651	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
840	140	5	46.7657	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
800	**140**	5	51.3265	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
800	**145**	5	52.8569	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
800	**150**	5	53.9366	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
800	**155**	5	54.7268	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
800	**160**	5	55.2657	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
800	140	**5**	38.9658	0.2500	1.0288	0.8515	0.2500	0.2355	1.5144	0.5814	0	0	0	0	0	0	0
800	140	**5.1**	35.7627	0.2500	1.0276	0.8515	0.2500	0.2361	1.5138	0.5810	0	0	0	0	0	0	0
800	140	**5.2**	32.8643	0.2500	1.0264	0.8515	0.2500	0.2367	1.5129	0.5806	0	0	0	0	0	0	0
800	140	**5.3**	29.3696	0.2500	1.0249	0.8515	0.2500	0.2374	1.5121	0.5802	0	0	0	0	0	0	0
800	140	**5.4**	26.1547	0.2500	**1.0230**	0.8515	0.2500	0.2384	1.5115	0.5799	0	0	0	0	0	0	0

Table-4.3: Values of α_1; β_1; τ_1; α_2; β_2; τ_2; α_3; β_3; ε_1; δ_1; ε_2; δ_2; ε_3; δ_3; Z_3 for changing values K_0; C_6; t and fixed values of other parameters:

Ko	C_6	t	Z_3	α_1	β_1	τ_1	α_2	β_2	τ_2	α_3	β_3	ε_1	δ_1	ε_2	δ_2	ε_3	δ_3
700	160	5	18.9876	0.2526	0.2121	0.0528	0.0000	0.6325	0.6280	0.2127	0.0713	0	0	0	0	0.0661	0.0007
710	160	5	16.8967	0.2551	0.2121	0.0425	0.0041	0.6221	0.5562	0.2416	0.0601	0	0	0	0	0.0581	0.0005
720	160	5	14.2316	0.2578	0.2121	0.0331	0.0256	0.6152	0.4999	0.2745	0.0511	0	0	0	0	0.0503	0.0003
730	160	5	12.7357	0.2589	0.2121	0.0284	0.0412	0.6045	0.4521	0.2887	0.0406	0	0	0	0	0.0412	0.0002
740	160	5	10.5793	0.2608	0.2121	0.0230	0.0653	0.5959	0.4265	0.3011	0.0347	0	0	0	0	0.0308	0.0001
700	**160**	5	9.0236	0.2526	0.2121	0.0528	0.0000	0.6325	0.6280	0.2127	0.0713	0	0	0	0	0.0661	0.0007
700	**165**	5	9.0241	0.2551	0.2121	0.0425	0.0041	0.6221	0.5562	0.2416	0.0601	0	0	0	0	0.0581	0.0005
700	**170**	5	9.0246	0.2578	0.2121	0.0331	0.0256	0.6152	0.4999	0.2745	0.0511	0	0	0	0	0.0503	0.0003
700	**175**	5	9.0250	0.2589	0.2121	0.0284	0.0412	0.6045	0.4521	0.2887	0.0406	0	0	0	0	0.0412	0.0002
700	**180**	5	9.0254	0.2608	0.2121	0.0230	0.0653	0.5959	0.4265	0.3011	0.0347	0	0	0	0	0.0308	0.0001
700	160	**5**	9.9876	0.2526	0.2121	0.0528	0.0000	0.6325	0.6280	0.2127	0.0713	0	0	0	0	0.0661	0.0007
700	160	**5.1**	8.7652	0.2522	0.2013	0.0425	0.0000	0.7545	0.7150	0.1520	0.0652	0	0	0	0	0.0446	0.0013
700	160	**5.2**	7.3245	0.2518	0.1921	0.0331	0.0001	0.8645	0.8452	0.0811	0.6020	0	0	0	0	0.0270	0.0019
700	160	**5.3**	6.9156	0.2516	0.1832	0.0284	0.0001	0.9223	0.8999	0.0259	0.5640	0	0	0	0	0.0123	0.0024
700	160	**5.4**	5.7834	0.2514	0.1709	0.0230	0.0002	0.9992	0.9510	0.0031	0.0545	0	0	0	0	0.0032	0.0031

Table-4.4: Values of α_1; β_1; τ_1; α_2; β_2; τ_2; α_3; β_3; ε_1; δ_1; ε_2; δ_2; ε_3; δ_3; Z_4 for changing values N_0; C_4; t and fixed values of other parameters:

No	C_4	t	Z_4	α_1	β_1	τ_1	α_2	β_2	τ_2	α_3	β_3	ε_1	δ_1	ε_2	δ_2	ε_3	δ_3
900	120	5	2.01E-09	0.2500	0.5985	0.3971	0.2423	0.4864	0.4864	0.3041	0.1944	0	0	0	0	0	0
910	120	5	1.99E-09	0.2500	0.5985	0.3971	0.2423	0.4864	0.4864	0.3041	0.1944	0	0	0	0	0	0
920	120	5	1.86E-09	0.2500	0.5985	0.3971	0.2423	0.4864	0.4864	0.3041	0.1944	0	0	0	0	0	0
930	120	5	1.73E-09	0.2500	0.5985	0.3971	0.2423	0.4864	0.4864	0.3041	0.1944	0	0	0	0	0	0
940	120	5	1.62E-09	0.2500	0.5985	0.3971	0.2423	0.4864	0.4864	0.3041	0.1944	0	0	0	0	0	0
900	**120**	5	2.09E-09	0.2500	0.5985	0.3971	0.2123	0.4864	0.4864	0.2500	0.1944	0	0	0	0	0	0
900	**125**	5	2.05E-09	0.2500	0.5985	0.3971	0.2123	0.4864	0.4864	0.2500	0.1944	0	0	0	0	0	0
900	**130**	5	2.01E-09	0.2500	0.5985	0.3971	0.2123	0.4864	0.4864	0.2500	0.1944	0	0	0	0	0	0
900	**135**	5	1.96E-09	0.2500	0.5985	0.3971	0.2123	0.4864	0.4864	0.2500	0.1944	0	0	0	0	0	0
900	**140**	5	1.92E-09	0.2500	0.5985	0.3971	0.2123	0.4864	0.4864	0.2500	0.1944	0	0	0	0	0	0
900	120	**5**	2.05E-09	0.2500	0.5985	0.3971	0.2123	0.4864	0.4864	0.2500	0.1944	0	0	0	0	0	0
900	120	**5.1**	2.03E-09	0.2500	0.5985	0.3847	0.2016	0.4743	0.4743	0.2500	0.1944	0	0	0	0	0	0
900	120	**5.2**	2.01E-09	0.2500	0.5985	0.3753	0.1926	0.4621	0.4621	0.2500	0.1944	0	0	0	0	0	0
900	120	**5.3**	1.98E-09	0.2500	0.5985	0.3612	0.1841	0.4552	0.4552	0.2500	0.1944	0	0	0	0	0	0
900	120	**5.4**	1.95E-09	0.2500	0.5985	0.3498	0.1711	0.4463	0.4463	0.2500	0.1944	0	0	0	0	0	0

Table-4.5: Values of α_1; β_1; τ_1; α_2; β_2; τ_2; α_3; β_3; ε_1; δ_1; ε_2; δ_2; ε_3; δ_3; Z_5 for changing values M_0; C_5; t and fixed values of other parameters:

Mo	C_5	t	Z_5	α_1	β_1	τ_1	α_2	β_2	τ_2	α_3	β_3	ε_1	δ_1	ε_2	δ_2	ε_3	δ_3
800	140	5	4.22E-10	0.2500	0.5520	0.4102	0.2483	0.4247	0.4247	0.3041	0.1416	0	0.6136	0	0	0	0
810	140	5	3.86E-10	0.2500	0.5311	0.3795	0.2383	0.4664	0.4664	0.3041	0.1416	0	0.6136	0	0	0	0
820	140	5	3.43E-10	0.2500	0.5120	0.3312	0.2213	0.5014	0.5014	0.3041	0.1416	0	0.6136	0	0	0	0
830	140	5	3.06E-10	0.2500	0.4899	0.3102	0.2103	0.5459	0.5459	0.3041	0.1416	0	0.6136	0	0	0	0
840	140	5	2.86E-10	0.2500	0.4612	0.2872	0.2028	0.5874	0.5874	0.3041	0.1416	0	0.6136	0	0	0	0
800	**140**	5	4.22E-10	0.2500	0.5520	0.4102	0.2483	0.4247	0.4247	0.3041	0.1416	0	0.6136	0	0	0	0
800	**145**	5	3.86E-10	0.2500	0.5311	0.0144	0.2383	0.4664	0.4664	0.3041	0.1416	0	0.6136	0	0	0	0
800	**150**	5	3.43E-10	0.2500	0.5120	0.0142	0.2213	0.5014	0.5014	0.3041	0.1416	0	0.6136	0	0	0	0
800	**155**	5	3.06E-10	0.2500	0.4899	0.0139	0.2103	0.5459	0.5459	0.3041	0.1416	0	0.6136	0	0	0	0
800	**160**	5	2.86E-10	0.2500	0.4612	0.0124	0.2028	0.5874	0.5874	0.3041	0.1416	0	0.6136	0	0	0	0
800	140	**5**	4.22E-10	0.2500	0.5520	0.4102	0.2483	0.4247	0.4247	0.3041	0.1416	0	0.6136	0	0	0	0
800	140	**6**	3.76E-10	0.2500	0.5311	0.0144	0.2383	0.4664	0.4664	0.3041	0.1416	0	0.6136	0	0	0	0
800	140	7	3.24E-10	0.2500	0.5120	0.0142	0.2213	0.5014	0.5014	0.3041	0.1416	0	0.6136	0	0	0	0
800	140	**8**	2.73E-10	0.2500	0.4899	0.0139	0.2103	0.5459	0.5459	0.3041	0.1416	0	0.6136	0	0	0	0
800	140	**9**	2.56E-10	0.2500	0.4612	0.0124	0.2028	0.5874	0.5874	0.3041	0.1416	0	0.6136	0	0	0	0

Table-4.6: Values of α_1; β_1; τ_1; α_2; β_2; τ_2; α_3; β_3; ε_1; δ_1; ε_2; δ_2; ε_3; δ_3; Z_6 for changing values K_0; C_6; t and fixed values of other parameters:

Ko	C_6	t	Z_6	α_1	β_1	τ_1	α_2	β_2	τ_2	α_3	β_3	ε_1	δ_1	ε_2	δ_2	ε_3	δ_3
700	160	5	4.22E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
710	160	5	3.86E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
720	160	5	3.43E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
730	160	5	3.06E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
740	160	5	2.86E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
700	**160**	5	4.22E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
700	**165**	5	3.86E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
700	**170**	5	3.43E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
700	**175**	5	3.06E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
700	**180**	5	2.86E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
700	160	**5**	4.22E-10	0.2346	0.6288	0.0157	0.2346	0.4378	0.3121	0.2346	0.1257	0	0.4691	0	0	0	0
700	160	**6**	3.76E-10	0.2402	0.6166	0.0144	0.2402	0.4378	0.3012	0.2402	0.1047	0	0.4691	0	0	0	0
700	160	7	3.24E-10	0.2580	0.5975	0.0142	0.2580	0.4378	0.2978	0.2580	0.8901	0	0.4691	0	0	0	0
700	160	**8**	2.73E-10	0.2870	0.5829	0.0139	0.2870	0.4378	0.2921	0.2870	0.6258	0	0.4691	0	0	0	0
700	160	**9**	2.56E-10	0.3870	0.5703	0.0124	0.3870	0.4378	0.2891	0.3870	0.4789	0	0.4691	0	0	0	0

V. DISCUSSION AND ANALYSIS

5.1. Observation of Optimal size of Bacterial units (Z_1) in Stage-I Plants with varying values of No, C_4, t:

From table-4.1, it is observed that both β_2 and τ_2 are equal and increasing functions; α_1, α_3, β_3, and δ_1 are invariant; and β_1, τ_1, α_2 are decreasing functions of No; the group of rates (α_1, α_2, α_3 and β_3) are less than the group of rates (β_1, δ_1 and β_2). As a result, the optimal (minimum) size of expected bacterial units in stage-I plants (Z_1) is a decreasing function of initial number of bacterial units (N_0) in stage-I. Further, with respect to upper limit on the variance of bacterial units (C_4), it is observed that both β_2 and τ_2 are equal and increasing functions; α_1, α_3, β_3, and δ_1 are invariant; and β_1, τ_1, α_2 are decreasing functions; the group of rates (α_1, α_2, α_3 and β_3) are less than the group of rates (β_1, δ_1 and β_2). As a result, the optimal (minimum) size of expected bacterial units (Z_1) is increasing function of C_4 in stage-I plants. It is also observed that, with the increased time β_2, τ_2 are equal and increasing functions; α_1, α_3 and β_3 are invariant; β_1, τ_1, α_2 are decreasing functions; δ_1 is increasing function of time; group of rates (α_1, α_2, α_3 and β_3) are less than the group of rates (β_1, δ_1 and β_2). As a result, the optimal (minimum) size of expected bacterial units (Z_1) is a decreasing function of time.

5.2. Observation of Optimal size of Bacterial units (Z_2) in Stage-II plants with varying values of Mo, C_5, t:

From table-4.2, it is observed that both α_1 and α_2 are equal; also β_1 is greater than all other rates except τ_2; all rates are invariant. As a result, the optimal (minimum) size of expected bacterial units (Z_2) on stage plants is an increasing function of initial number of bacterial units (M_0) in stage-II plants when other parameters are constants. All birth, death and transition rates of bacteria in stage-I plants are not influenced by (M_0). Hence it may conclude that as the initial number of bacterial units in stage-II plants is increasing, the expected size of these bacteria is also increasing as the birth and death rates are constant. With respect to the upper limit of variance of bacteria on stage-II plants, it is observed that both α_1 and α_2 are equal; β_1 is greater than all other rates except τ_2; all the rest of the rates are invariant. As a result, the optimal (minimum) size of expected bacterial units (Z_2) is an increasing function in stage-II plants when other parameters are constants. All birth, death and transition rates of bacteria in stage-I are not influenced by C_5. Hence it may conclude that as the variance of bacterial units in stage-II plants is increasing, the expected size of bacteria on stage-II plants is also increasing as the birth and death rates are constant. Regarding the change of time duration, it is observed that both α_1 and α_2 are equal; β_1 is greater than all other rates except τ_2; α_3, β_1 and τ_2 are decreasing functions; β_2 is increasing function; all the remaining rates are invariant. As a result, the optimal (minimum) size of expected bacterial units (Z_2) is a decreasing function of time when other parameters are constants. Further, birth rate (β_1) of bacteria in stage-I plants is decreasing function of time where as birth rate (β_2) in stage-II plants and arrival rate (α_3) in stage-III plants are increasing and decreasing functions of time respectively. Hence it may conclude that as the time elapsed, the expected size of the bacteria in stage-II plants is decreasing.

5.3. Observation of Optimal size of Bacterial units (Z_3) in Stage-III plants with varying values of Ko, C_6, t:

From table-4.3, it is observed that α_1, α_2 and α_3 are increasing functions; β_1 is invariant; τ_1, β_2, τ_2, β_3, ε_3 and δ_3 are decreasing functions of (Ko); As a result, the optimal (minimum) size of expected bacterial units (Z_3) is a decreasing function of initial number of bacterial units (K_0) in stage-III plants. Regarding with upper limit on the variance of stage-III plants(C_6), α_1, α_2 and α_3 are increasing functions; also β_1 is invariant; τ_1, β_2, τ_2, β_3, ε_3 and δ_3 are decreasing functions. As a result, the optimal (minimum) size of expected bacterial units (Z_3) is an increasing function of C_6. Hence it may conclude that as the variance number of bacterial units in stage-III plants is increasing, the expected size of these bacteria is also increasing. With respect to time period observation, α_1, β_1, τ_1, α_3, β_3 and ε_3 are decreasing functions; α_2, β_2, τ_2, and δ_3 are increasing functions. As a result, the optimal (minimum) size of expected bacterial units (Z_3) is a decreasing function of time period when other parameters are constants. Hence it may conclude that as the time elapsed, the expected size of bacteria in stage-III plants is decreasing.

5.4. Observation of Optimal Variance of Bacterial units (Z_4) in Stage-I Plants with varying values of No, C_4, t:

From table-4.4, with respect to the initial size of bacteria on the stage-I plant, it is observed that all the birth, death and transition rates are invariant; the group of rates (α_1, α_2, α_3 and β_3) are less than the group of rates (β_1, and β_2). As a result, the optimal (maximum) size of variance of bacterial units (Z_4) in stage-I plants is a decreasing function of N_0. Hence it may conclude that as the initial number of bacterial units in stage-I plants is increasing, the volatility of these bacteria is

decreasing as the birth and death rates are nullified in stage-I plants. Regarding the change in upper limit on variance of bacterial units (C_4) in stage-I plants, the group of rates ($\alpha_1, \alpha_2, \alpha_3, \beta_3$) are less than the group of rates ($\beta_1, \tau_1, \beta_2, \tau_2$); all the rates are invariant. As a result, the optimal (maximum) size of variance of bacterial units (Z_4) in stage-I plants is a decreasing function of upper limit of variance of bacterial units (C_4) in stage-I plants when other parameters are constants. Hence it may conclude that as the upper limit of variance of bacterial units (C_4) in stage-I plants is increasing, the volatility of these bacteria is decreasing due to nullified arrival, birth and death rates in stage-I plants. With respect to increase in the time duration the group of rates ($\alpha_1, \alpha_2, \alpha_3, \beta_3$) are less than ($\beta_1, \tau_1, \beta_2, \tau_2$) as well as the group of rates ($\tau_1, \alpha_2, \beta_2, \tau_2$); are decreasing functions. As a result, the optimal (maximum) size of variance of bacterial units (Z_4) in stage-I plants is a decreasing function of time when other parameters are constants. Hence it may conclude that as the time is elapsed, the volatility of these bacteria is decreasing due to nullified arrival, birth and death rates in stage-I.

5.5. Observation of Optimal Variance of Bacterial units (Z_5) in Stage-II plants with varying values of Mo, C_5, t:

From table-4.5, regarding the increased values of initial number of bacteria in stage-II plants, the group of rates ($\alpha_1, \alpha_2, \alpha_3, \beta_3$) are less than the group of rates ($\beta_1, \tau_1, \beta_2, \tau_2$); τ_1, α_2 are decreasing functions; β_2, τ_2 are increasing functions. As a result, the optimal (maximum) size of variance of bacterial units (Z_5) in stage-II plants is decreasing function of M_0 when other parameters are constants. Hence it may conclude that for the increasing initial number of bacterial units in stage-II plants there will be decrease in the volatility of bacteria in stage-II plants. With related to increase in upper limit of variance of bacterial units in stage-II plants (C_5), it is observed that the group of rates ($\alpha_1, \alpha_2, \alpha_3, \beta_3$) are less than the group of rates (β_1, β_2, τ_2); τ_1, α_2 are decreasing functions; β_2, τ_2 are increasing functions. As a result, the optimal (maximum) size of variance of bacterial units (Z_5) in stage-II plants is decreasing function of C_5 when other parameters are constants. Hence it may conclude that the volatility of bacteria on stage –II plants is decreasing for the increasing C_5. Further, with regarding to time period of study, it is observed that the group of rates ($\alpha_1, \alpha_2, \alpha_3, \beta_3$) are less than the group of rates (β_1, β_2, τ_2); τ_1, α_2 are decreasing functions; β_2, τ_2 are increasing functions. As a result, the optimal (maximum) size of variance of bacterial units (Z_5) in stage-II plants is decreasing function of time when other parameters are constants. Hence it may conclude that for the increasing time, the volatility of these bacteria is decreasing.

5.6. Observation of Optimal Variance of Bacterial units (Z_6) in Stage-III plants with varying values of Ko, C_6, t:

From table-4.6, with respect to the increased value of initial number of bacterial units on the stage-III plants (K_0), it is observed that the group of rates ($\alpha_1, \alpha_2, \alpha_3, \beta_3, \tau_1$) are less than the group of rates (β_1, β_2, τ_2); all rates are invariant. As a result, the optimal (maximum) size of variance of bacterial units (Z_6) in stage-III plants is decreasing function of K_0 when other parameters are constants. Hence it may conclude that for the increasing K_0, the volatility of bacteria is decreasing. Regarding the change in upper limit on the variance of bacterial units on stage-III plants (C_6), the group of rates ($\alpha_1, \alpha_2, \alpha_3, \beta_3, \tau_1$) are less than ($\beta_1, \beta_2, \tau_2$); all rates are invariant. As a result, the optimal (maximum) size of variance of bacterial units (Z_6) in stage-III plants is decreasing function of C_6 when other parameters are constants. Hence it may conclude that for the increasing C_6, the volatility of the bacteria is decreasing. Regarding the change of time duration, ($\alpha_1, \alpha_2, \alpha_3, \beta_3, \tau_1, \tau_2$) are less than ($\beta_1, \beta_2$); β_1, τ_1, τ_2 are decreasing functions $\alpha_1, \alpha_2, \alpha_3, \beta_3$ are increasing functions. As a result, the optimal (maximum) size of variance of bacterial units (Z_6) in stage-III plants is decreasing function of time period when other parameters are constants.

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